Chapter 5 - Practice Problems 2

Determine the possible values of the random variable.

1) Suppose that two balanced dice are rolled. Let \( Y \) denote the product of the two numbers. What are the possible values of the random variable \( Y \)?

2) Suppose that two balanced dice, a red die and a green die, are rolled. Let \( Y \) denote the value of \( G - R \) where \( G \) represents the number on the green die and \( R \) represents the number on the red die. What are the possible values of the random variable \( Y \)?

Use random-variable notation to represent the event.

3) Suppose a coin is tossed four times. Let \( X \) denote the total number of tails obtained in the four tosses. Use random-variable notation to represent the event that the total number of tails is three.

4) Suppose that two balanced dice are rolled. Let \( Y \) denote the product of the two numbers. Use random-variable notation to represent the event that the product of the two numbers is greater than 4.

5) The following table displays a frequency distribution for the number of siblings for students in one middle school. For a randomly selected student in the school, let \( X \) denote the number of siblings of the student.

<table>
<thead>
<tr>
<th>Number of siblings</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>189</td>
<td>245</td>
<td>102</td>
<td>42</td>
<td>24</td>
<td>13</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Use random-variable notation to represent the event that the student obtained has fewer than two siblings.

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Provide an appropriate response.

6) A coin is biased. Danny wishes to determine the probability of obtaining heads when flipping this coin. He flips the coin 10 times and obtains 8 heads. He concludes that the probability of obtaining heads when flipping this coin is 0.8. Is his thinking reasonable? Why or why not?
Obtain the probability distribution of the random variable.

7) When a coin is tossed four times, sixteen equally likely outcomes are possible.

Let $X$ denote the total number of tails obtained in the four tosses. Find the probability distribution of the random variable $X$. Leave your probabilities in fraction form.

A) $x P(X = x)$  
0 $\frac{1}{16}$  
1 $\frac{3}{16}$  
2 $\frac{1}{2}$  
3 $\frac{3}{16}$  
4 $\frac{1}{16}$

B) $x P(X = x)$  
0 $\frac{1}{16}$  
1 $\frac{3}{8}$  
2 $\frac{3}{8}$  
3 $\frac{1}{4}$  
4 $\frac{1}{16}$

C) $x P(X = x)$  
1 $\frac{1}{4}$  
2 $\frac{7}{16}$  
3 $\frac{1}{4}$  
4 $\frac{1}{16}$  
5 $\frac{1}{4}$

D) $x P(X = x)$  
0 $\frac{1}{16}$  
1 $\frac{1}{4}$  
2 $\frac{3}{8}$  
3 $\frac{1}{4}$  
4 $\frac{1}{16}$

8) When two balanced dice are rolled, 36 equally likely outcomes are possible.

Let $X$ denote the smaller of the two numbers. If both dice come up the same number, then $X$ equals that common value. Find the probability distribution of $X$. Leave your probabilities in fraction form.

A) $x P(X = x)$  
1 $\frac{1}{6}$  
2 $\frac{1}{6}$  
3 $\frac{1}{6}$  
4 $\frac{1}{6}$  
5 $\frac{1}{6}$  
6 $\frac{1}{6}$

B) $x P(X = x)$  
1 $\frac{5}{18}$  
2 $\frac{2}{9}$  
3 $\frac{1}{6}$  
4 $\frac{1}{6}$  
5 $\frac{1}{6}$  
6 $\frac{1}{6}$

C) $x P(X = x)$  
1 $\frac{5}{18}$  
2 $\frac{1}{4}$  
3 $\frac{7}{36}$  
4 $\frac{5}{36}$  
5 $\frac{1}{12}$  
6 $\frac{1}{36}$

D) $x P(X = x)$  
1 $\frac{11}{36}$  
2 $\frac{1}{4}$  
3 $\frac{7}{36}$  
4 $\frac{5}{36}$  
5 $\frac{1}{12}$  
6 $\frac{1}{36}$

Find the specified probability.

9) There are only 8 chairs in our whole house. Whenever there is a party some people have no where to sit. The number of people at our parties (call it the random variable $X$) changes with each party. Past records show that the probability distribution of $X$ is as shown in the following table. Find the probability that everyone will have a place to sit at our next party.

<table>
<thead>
<tr>
<th>$x$</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>&gt;10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.20</td>
<td>0.15</td>
<td>0.15</td>
<td>0.10</td>
<td>0.30</td>
</tr>
</tbody>
</table>

A) 0.45  
B) 0.15  
C) 0.05  
D) 0.55

10) Use the special addition rule and the following probability distribution to determine $P(6 < X \leq 8)$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.20</td>
<td>0.15</td>
<td>0.15</td>
<td>0.10</td>
<td>0.30</td>
</tr>
</tbody>
</table>

A) 1.00  
B) 0.40  
C) 0.45  
D) 0.35

Calculate the specified probability

11) Suppose that $W$ is a random variable. Given that $P(W \leq 4) = 0.825$, find $P(W > 4)$.

A) 0.825  
B) 0  
C) 0.175  
D) 4

12) Suppose that $K$ is a random variable. Given that $P(-3.2 \leq K \leq 3.2) = 0.175$, and that $P(K < -3.2) = P(K > 3.2)$, find $P(K > 3.2)$.

A) 1.6  
B) 0.175  
C) 0.4125  
D) 0.825
Construct the requested histogram.

13) If a fair coin is tossed 4 times, there are 16 possible sequences of heads (H) and tails (T). Suppose the random variable $X$ represents the number of heads in a sequence. Construct the probability distribution for $X$.

A) ![Histogram A]  
B) ![Histogram B]  
C) ![Histogram C]  
D) ![Histogram D]

Provide an appropriate response.

14) The random variable $X$ represents the number of siblings of a student selected randomly from a particular college. Use random variable notation to express the following statement in shorthand.

The probability that the student has two siblings is 0.18.

A) $P(X) = 0.18$  
B) $P(X = 2) = 0.18$  
C) $P(2) = 0.18$  
D) $(X = 2) = 0.18$

Find the mean of the random variable.

15) The random variable $X$ is the number of houses sold by a realtor in a single month at the Sendsom's Real Estate office. Its probability distribution is given in the table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(X = x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.24</td>
</tr>
<tr>
<td>1</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>0.16</td>
</tr>
<tr>
<td>4</td>
<td>0.01</td>
</tr>
<tr>
<td>5</td>
<td>0.14</td>
</tr>
<tr>
<td>6</td>
<td>0.11</td>
</tr>
<tr>
<td>7</td>
<td>0.21</td>
</tr>
</tbody>
</table>

A) 3.50  
B) 3.40  
C) 3.60  
D) 3.35

Find the expected value of the random variable.

16) In a game, you have a $\frac{1}{47}$ probability of winning $150 and a $\frac{46}{47}$ probability of losing $2. What is your expected value?

A) $1.23$  
B) $-1.96$  
C) $5.15$  
D) $3.19$
17) Suppose you buy 1 ticket for $1 out of a lottery of 1,000 tickets where the prize for the one winning ticket is to be $500. What is your expected value?

A) $0.50  B) $0.40  C) $1.00  D) $0.00

Find the standard deviation of the random variable.

18) The probabilities that a batch of 4 computers will contain 0, 1, 2, 3, and 4 defective computers are 0.4979, 0.3793, 0.1084, 0.0138, and 0.0007, respectively.

A) 0.54  B) 0.97  C) 0.73  D) 0.68

The probability distribution of a random variable is given along with its mean and standard deviation. Draw a probability histogram for the random variable; locate the mean and show one, two, and three standard deviation intervals.

19) \[ \begin{array}{cccccc}
 x & 4 & 5 & 6 & 7 & 8 \\
P(X = x) & 0.1 & 0.3 & 0.45 & 0.1 & 0.05 \\
\end{array} \]

A) \[ \mu = 5.7, \sigma = 0.95 \]

B) \[ \mu = 5.7, \sigma = 0.95 \]

C) \[ \mu = 5.7, \sigma = 0.95 \]
Answer Key
Testname: CH 5 SET 2

1) D
2) C
3) D
4) A
5) C
6) Answers will vary. Possible answer: No, Danny's thinking is not reasonable. If Danny flipped the coin a large number of times, the proportion of heads would approximate the probability of obtaining heads. However, the number of observations here is too small.
7) D
8) D
9) A
10) D
11) C
12) C
13) A
14) B
15) C
16) A
17) A
18) C
19) B